Exploring the Use of Nonconventional Receivers for Quantum Communications

Joás J.M. Lacerda[1*], Leonardo J. Pereira[1], Lucas F. Melo[1], Wagner C. Normando Filho[1], Alexandre B. Tacla[1]
[1]*SENAI CIMATEC University - QuIIN - Quantum Industrial Innovation, EMBRAPII CIMATEC Competence Center in Quantum Technologies; Salvador, Bahia, Brazil*

Standard protocols in optical communication systems employ coherent states of light as the information carriers. The quantum nature inherent to these states introduces a fundamental complexity into the decoding process. In particular, when dealing with attenuated signals, the overlap between different states precludes the possibility of perfectly discriminate them. In this study, we investigate the performance of conventional and nonconventional quantum discrimination strategies under different informational metrics. We focus our analysis on binary coherent states within the ideal case of a lossless channel. The usual homodyne receiver was studied in comparison with the Kennedy and the optimized displacement receiver. We find that nonconventional strategies employing non-Gaussian measurements surpass the conventional homodyne discrimination scheme according to measurement error probability and mutual information.
Keywords: Quantum Communications. Quantum State Discrimination. Mutual Information. Non-Gaussian Measurement.
Abbreviations: POVM, Positive Operator-Valued Measure. BPSK, Binary Phase-Shift-Keying.

In general terms, the main objective of any communication system is to transmit information reliably from a transmitter to a receiver [1]. In optical quantum communication, it becomes necessary to employ the mathematical framework of quantum mechanics to adequately describe the transmission and reception of optical fields under various conditions [2]. This quantum description becomes essential in the photon-starved-regime, where the quantum shot noise fundamentally limits signal discrimination capabilities [3]. Under such circumstances, the detection process becomes a discrimination problem, and the theoretical framework of quantum state discrimination must be employed at the receiver's end of the communication protocol.

In this study, we aim to investigate different quantum state discrimination strategies by employing two distinct criteria: measurement error probability and mutual information. Our objective is to compare conventional and nonconventional reception strategies. We focus on binary coherent state modulation within the ideal case of a lossless channel.

Received on 16 September 2025; revised 21 November 2025.
Address for correspondence: Joás J. M. Lacerda. Av. Orlando Gomes, 1845 - Piatã, Salvador – BA – Brazil, Zipcode: 41650-010. E-mail: joas.lacerda@fbter.org.br.

J Bioeng. Tech. Health 2025;8(6):564-570

Fundamentals of Quantum Detection and Communication

In quantum theory, a physical system is represented by a positive semidefinite unit trace operator $\hat{\rho}_i$ on the system's Hilbert space $\mathscr{H}_s$, called a density operator [4]. When only pure states are assumed, its description is given by state vectors $\{|\psi_i\rangle\}$ as

$$\hat{\rho}_i = |\psi_i\rangle\langle\psi_i|. \quad (1)$$

For a quantum communication protocol, the transmitter, Alice, encodes a classical symbol a_0 or a_1 into two quantum states $|\psi_0\rangle$ and $|\psi_1\rangle$ with a priori probabilities p_0 and p_1, respectively. In the case of quantum optical communication, the usual information carrier are optical signals described by coherent states [1]. Here, we consider pure coherent states of the form

$$|\nu\rangle = \hat{D}(\nu)|0\rangle, \quad (2)$$

where $\hat{D}(\nu)$ is the displacement operator and $|0\rangle$ is the vaccum state [5,6]. In particular, our focus will be on analyzing a binary phase-shift-keying

(BPSK) configuration, characterized by the two quantum coherent states $|\alpha\rangle$ and $|-\alpha\rangle$. These states possess an identical amplitude magnitude $|\alpha|$; however, they are differentiated by a phase shift of π.

Once the signal is encoded, it is propagated through a quantum channel, described by a completely positive trace preserving map $\hat{G}$ [7], until it reaches the receiver, Bob, in the form of an altered state $\hat{G}(|\alpha_i\rangle)$. At this point, Bob infers by the incoming signal whether the transmitted symbol was a_0 or a_1. This inference is realized by performing a quantum measurement process, denoted by a positive-operator valued measurement (POVM) $\{\hat{\Pi}_{b_i}\}$

$$\sum_i \hat{\Pi}_{b_i} = \mathbb{1}, \qquad \hat{\Pi}_{b_i} \geq 0, \tag{3}$$

which retrieves an outcome bi correlated with the variables $\{a_0, a_1\}$ [4].

Hereafter we consider only the ideal case of a lossless channel, i.e. $\hat{G} = \mathbb{1}$, such that $\hat{G}(|\alpha_i\rangle) = |\alpha_i\rangle$. In this case, the communication procedure is depicted schematically on Figure 1.

Figure 1. Alice encodes symbol $a_0(a_1)$ into state $|-\alpha\rangle(|\alpha\rangle)$, which is then transmitted to Bob. Afterwards, Bob performs a measurement $\Pi_0(\Pi_1)$ that yields an outcome $b_0(b_1)$, which is correlated with the symbols encoded by Alice.

$$a_0 \longrightarrow |-\alpha\rangle \longrightarrow \hat{\Pi}_0 \longrightarrow b_0$$

$$a_1 \longrightarrow |\alpha\rangle \longrightarrow \hat{\Pi}_1 \longrightarrow b_1$$

With that, the conditional probability that Bob measures a result b_i given that Alice sent a signal a_j is given by the Born rule:

$$p(b_i|a_j) = \mathrm{Tr}(\hat{\Pi}_i|\alpha_j\rangle\langle\alpha_j|), \quad i = 0, 1; \tag{4}$$

with $|\alpha_0\rangle = |-\alpha\rangle \qquad |\alpha_1\rangle = |\alpha\rangle$.

Two general metrics are often used to evaluate the efficiency of a communication system [8]; one is the error probability defined, in our case, as Weedbrook and colleagues [9]:

$$P_{\mathrm{err}} = p_0\mathrm{Tr}(\hat{\Pi}_1|\alpha_0\rangle\langle\alpha_0|) + p_1\mathrm{Tr}(\hat{\Pi}_0|\alpha_1\rangle\langle\alpha_1|). \tag{5}$$

The other is the mutual information defined as Helstrom [10]:

$$I(A:B) = H(A) - H(A|B), \tag{6}$$

which measures the amount of information about Alices's variable extracted by Bob for a particular measurement [11]. In that expression $H(A) = -\sum_a p(a)\log_2(a)$ and $H(A|B) = \sum_b p(b)H(A|B = b)$ are the usual and conditional Shannon entropies.

Coherent State Discrimination

Because of the nature of the quantum states used in quantum communication protocols, the classical values decoded by Bob can differ from the ones encoded by Alice, even for a lossless channel. This limitation arises from the quantum trace of those states, and it is related to the impossibility of perfectly distinguishing between two non-orthogonal states [12]. For the case of coherent states, this non-orthogonality is closely associated with the inherent quantum shot noise characteristic of such states. This relationship manifests as an overlap in the probability regions within the phase-space representation (Figure 2).

This quantum state discrimination problem is ultimately a measurement optimization issue, that is, the best strategy for determining the state received must be given by the POVM that minimizes the probability of error of equation (5). The minimum error probability allowed by the quantum theory is given by the Helstrom limit, which for binary signals of equal a priori probabilities is given by [9]

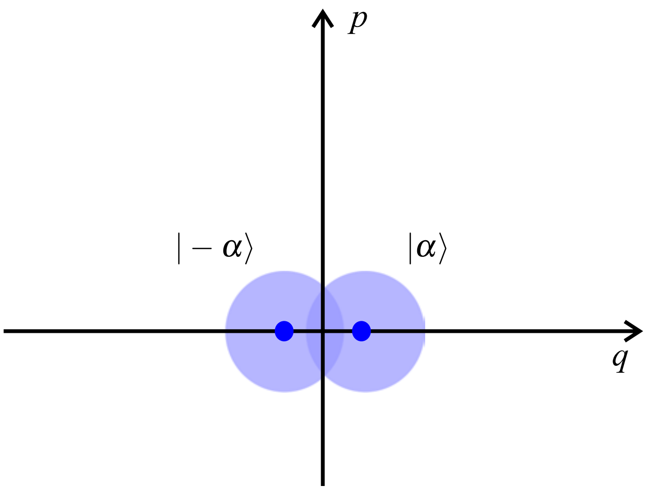
$$P_{\mathrm{Hel}} = \frac{1}{2}\left[1 - \sqrt{1 - e^{-4|\alpha|^2}}\right]. \tag{7}$$

Implementing the optimal POVM as proposed by Helstrom presents substantial complexity when applied to coherent states [13]. Nevertheless, there exist feasible discrimination techniques that can approximate the optimal POVM, which brings us closer to achieving the Helstrom limit.

Conventional and Non-Conventional Receivers

In the literature on quantum state discrimination, each discrimination scheme defines a receiver.

Figure 2. Representation of states $|\alpha\rangle$ and $|-\alpha\rangle$ in phase space. The intrinsic quantum noise is responsible for overlap between states.



Generally speaking, the structure of a receiver comprises an unitary operation, which is succeeded by a detection process [14]. The usual schemes applied for coherent state discrimination are based on Gaussian detections, that is, the possible outcomes follows a gaussian distribution [15]. In the context of optical communication with BPSK modulation, these detections are centered on the measurement of field quadrature components through a homodyne detection technique [9, 16].

Nonetheless, non-Gaussian measurements hold considerable significance in specific protocols, as they have the potential to surpass conventional detection methods concerning figures of merit that are pertinent to the field of quantum communication [17-19]. In particular, non-Gaussian on/off detections have been used as an essential tool in the context of coherent state discrimination [20-23]. In the following, we shall examine two main discrimination schemes that implement non-Gaussian detection, referred to as non- conventional receivers, and conduct a comparative analysis with the conventional homodyne receiver.

Discrimination Strategies

A homodyne receiver is a discrimination strategy based on the homodyne detection, consisting exclusively of Gaussian operations. The POVM characterizing this receiver is formulated as

$$\hat{\Pi}_0 = \int_{-\infty}^{0} |x\rangle\langle x|\, dx, \qquad \hat{\Pi}_1 = \mathbb{1} - \hat{\Pi}_0. \quad (8)$$

Here, $(|x\rangle\langle x|)$ denotes a projection operator that projects onto the eigenstates of the quadrature op- erator $\hat{x}$. Consequently, when a measurement of the quadrature x yields a positive outcome, it is indicative of the state $|\alpha\rangle$. In contrast, negative results identify the state $|-\alpha\rangle$.

The equation (5) together with the POVM elements above gives the error probability of the homodyne receiver:

$$P_H = \frac{1}{2}\left[1 - \mathrm{erf}(\sqrt{2}|\alpha|)\right], \quad (9)$$

where $\mathrm{erf}(x)$ is the error function. This error probability is also known as the *Gaussian limit*. This is due to the fact that among all Gaussian measurements, homodyne detection represents the optimal approach for the discrimination of binary coherent states [17]. This approach approximates the Helstrom limit, being nearly optimal, for coherent states characterized by a very low mean photon number $|\alpha|^2$. However, the effectiveness of this strategy diminishes, as $|\alpha|^2$ increases.

The first proposal for a practical discrimination scheme of binary coherent states considered near-optimum, was formulated by R. Kennedy [20]. Its receiver, also known as nulling-displacement receiver, consists first of applying a displacement operation with amplitude α in both states such that

$$|-\alpha\rangle \longrightarrow |0\rangle \qquad |\alpha\rangle \longrightarrow |2\alpha\rangle. \quad (10)$$

Then, its applied an on/off detection on the displaced state to measure the presence of any photons. This detection is represented by the operators

$$\hat{\Pi}_0 = |0\rangle\langle 0|; \quad \hat{\Pi}_1 = \mathbb{1} - \hat{\Pi}_0 = \sum_{n=1}^{\infty} |n\rangle\langle n|, \quad (11)$$

where $|n\rangle\langle n|$ are projectors into Fock states, i.e., states of a well-defined number of photons [5].

The principle here is to map the problem of discriminating between $|-\alpha\rangle$ and $|\alpha\rangle$ into discriminating between vacuum $|0\rangle$ and state $|2\alpha\rangle$. This strategy minimizes the first term of equation (5) and the error probability is given by

$$P_{\mathrm{K}} = \frac{1}{2} e^{-4\alpha^2}. \tag{12}$$

Although we have a value higher than the Helstrom limit, $P_{\mathrm{K}} > P_{\mathrm{Hel}}$, the Kennedy receiver is classified as *near-optimum*, since in the high-energy regime we have $P_{\mathrm{K}} \approx 2P_{\mathrm{Hel}}$ [24]. Consequently, this strategy surpasses the performance of the homodyne receiver for most of the energy values of the coherent states. However, when dealing with highly attenuated signals, the homodyne receiver constitutes a more effective method of discrimination. This behavior is illustrated in Figure 3, which presents the error probabilities associated with the homodyne and Kennedy receivers against the Helstrom limit, depicted as a function of the mean photon number.

An optimized the displacement parameter of the Kennedy receiver can overcome the advantage a homodyne receiver holds in scenarios characterized by low-amplitude signals. This strategy, first proposed by Takeoka and colleagues [17], is known as optimized displecement receiver and consists of determining the displacement parameter β that minimizes the error probability for each amplitude of the coherent state. In this case, before on/off detection, both states are displaced by an optimized factor β such that

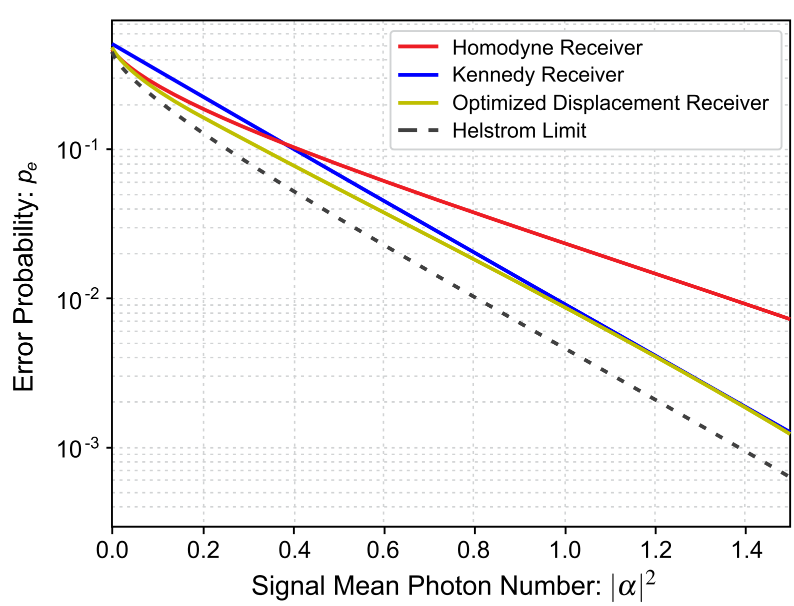
$$|\pm\alpha\rangle \longrightarrow \hat{D}(\beta)\,|\pm\alpha\rangle = |\pm\alpha+\beta\rangle\,. \tag{13}$$

The error probability for this receiver is given by

$$P_{\beta} = \frac{1}{2}\left(e^{-(\beta+\alpha)^2} - e^{-(\beta-\alpha)^2} + 1\right). \tag{14}$$

As shown in Figure 3, the displacement optimization reduces the probability of error compared to the homodyne receiver for any intensity value, including, in particular, signals with a low mean photon number. For high energy values, we have an expected convergence $P_{\beta} \to P_{\mathrm{K}}$. The practical implementation of this receiver has been demonstrated experimentally through different studies [22,25]. Furthermore, this strategy presents possible applications in quantum key distribution, where it has been shown to increase the secret key rate compared to that of a simple homodyne detection [18].

Figure 3. Logarithmic plot of error probability as a function of |α|² for the optimized displacement receiver (solid yellow curve), Kennedy receiver (solid blue curve), homodyne receiver (solid red curve), and the Helstrom limit (dashed black curve).



Mutual Information and Discrimination Strategies

Another fundamental criterion for evaluating the performance of a communication system is the mutual information between the random variables associated with the transmitter and the receiver. Here we describe the behavior of mutual information for the different receivers presented in the last section. When considering a lossless channel and a priori probabilities $p(a_j) = 1/2$, the equation (6) can be worked on to become a function of the conditional probabilities:

$$I(A:B) = 1 + \frac{1}{2}\sum_{k=0}^{1}\sum_{j=0}^{1} p(b_k|a_j)\log_2 \frac{p(b_k|a_j)}{\sum_l p(b_k|a_l)}. \tag{15}$$

With this equation, we can directly evaluate the mutual information through the POVM associated with each receiver and the Born rule equation (4).

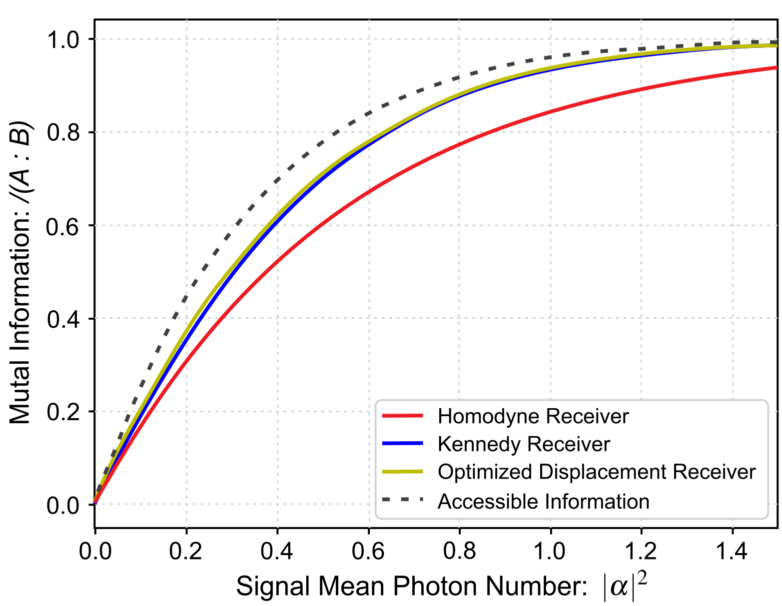
The numerical results obtained for the different receivers are presented in Figure 4 as a function of the mean number of photons of the signal. For comparison is also plotted the accessible information for the BPSK modulation, obtained by the maximization of the mutual information over all possible POVM's: $\max_{\hat{\Pi}_b} I(A:B)$.

In Figure 4, it is easily seen how both nonconventional receivers outperform simple homodyne detection, even for low-intensity signals. In addition, the mutual information of the nonconventional receivers saturates more rapidly than the homodyne one that demands a higher signal energy to reach the maximum. However, when comparing both nonconventional receivers, significantly less contrast is observed. Although the optimized displacement receiver demonstrates a similar trend to the error probability — exceeding the performance of the Kennedy receiver for attenuated signals and converging at higher energy values — the distinction is subtle, evidencing only a slight advantage for low-energy signals. For higher energy both strategies saturates equally and in general the mutual information is comparable.

Conclusion

In this work, we calculated the error probability and mutual information considering different

Figure 4. Plot of the mutual information as a function of |α|² for the optimized displacement receiver (solid yellow curve), Kennedy receiver (solid blue curve), homodyne receiver (solid red curve). For comparison the accessible information (dashed black curve) is also plotted.



discrimination strategies for binary coherent states. As a result, we show that, in particular, the optimized displacement receiver outperformed the homodyne receiver for both criteria considered. The Kennedy receiver surpasses the mutual information achieved by the conventional discrimination strategy across all energy levels; however, when analyzing the error probability, it is demonstrated to be a less effective strategy for highly attenuated signals. Furthermore, Kennedy receiver shows considerable improvement in the error probability when optimized displacement is implemented. However, the mutual information derived from these two strategies remains comparable at all energy levels.

The gap between the error probabilities and the Helstrom limit, as well as the discrepancy between the calculated mutual information and the accessible information, indicates the potential for further development. In addition, the analysis of discrimination state strategies through communication metrics opens the possibility of further studies of applications in fields related to quantum information processing. In particular, improvement of the mutual information between trusted parties is a critical aspect in quantum key distribution, such that a direct problem related to the optimization of measurements can be explored from the standpoint of the secret-key rate.

Acknowlegement

This work has been fully funded by the project ‘Nonconventional Receivers in CV-QKD’ supported by QuIIN - Quantum Industrial Innovation, EMBRAPII CIMATEC Competence Center in Quantum Technologies, with financial resources from the PPI IoT/Manufatura 4.0 of the MCTI grant number 053/2023, signed with EMBRAPII.

References

1. Michele N. Notarnicola. Quantum communications in continuous variable systems. Phd thesis, Università Degli Studi di Milano, 2024.
2. J. Gordon. Quantum effects in communications systems. Proceedings of the IRE, 50(9):1898–1908, sep 1962.
3. K. Banaszek, L. Kunz, M. Jachura, and M. Jarzyna. Quantum limits in optical communications. Journal of Lightwave Technology, 38(10):2741–2754, may 2020.
4. S. Barnett. Quantum Information. Oxford Master Series in Physics. OUP Oxford, 2009.
5. L. Mandel and E. Wolf. Optical Coherence and Quantum Optics. EBL-Schweitzer. Cambridge University Press, 1995.
6. Marlan O. Scully and M. Suhail Zubairy. Quantum Optics. Cambridge University Press, 1997.
7. J.A. Bergou and M. Hillery. Introduction to the Theory of Quantum Information Processing. Graduate Texts in Physics. Springer New York, 2013.
8. K. Kato, M. Osaki, M. Sasaki, and O. Hirota. Quantum detection and mutual information for qam and psk signals. IEEE Transactions on Communications, 47(2):248–254, 1999.
9. Christian Weedbrook, Stefano Pirandola, Raúl García-Patrón, Nicolas J. Cerf, Timothy C. Ralph, Jeffrey H. Shapiro, and Seth Lloyd. Gaussian quantum information. Rev. Mod. Phys., 84:621–669, May 2012.
10. M.A. Nielsen and I.L. Chuang. Quantum Computation and Quantum Information: 10th Anniversary Edition. Cambridge University Press, 2010.
11. Michele N. Notarnicola and Stefano Olivares. Employing weak-field homodyne detection for optical communications. IEEE Journal on Selected Areas in Communications, 43(5):1524–1535, 2025.
12. Carl W. Helstrom. Quantum detection and estimation theory. Journal of Statistical Physics, 1(2):337–354, 1969.
13. C. W. Helstrom. Quantum Detection and Estimation Theory. Elsevier, Academic Press, New York, 1976.
14. Masahide Sasaki and Osamu Hirota. Optimum decision scheme with a unitary control process for binary quantum-state signals. Phys. Rev. A, 54:2728–2736, Oct 1996.
15. I. B. Djordjevic. Quantum Communication, Quantum Networks, and Quantum Sensing. Academic Press, London, 2022.
16. S. L. Braunstein and P. van Loock. Quantum information with continuous variables. Reviews of Modern Physics, 77(2):513–577, may 2005.
17. Masahiro Takeoka and Masahide Sasaki. Discrimination of the binary coherent signal: Gaussian-operation limit and simple non-gaussian near-optimal receivers. Phys. Rev. A, 78:022320, Aug 2008.
18. Christoffer Wittmann, Ulrik L. Andersen, Masahiro Takeoka, Denis Sych, and Gerd Leuchs. Demonstration of coherent-state discrimination using a displacement-controlled photon-number-resolving detector. Phys. Rev. Lett., 104:100505, Mar 2010.

19. Shuro Izumi, Masahiro Takeoka, Mikio Fujiwara, Nicola Dalla Pozza, Antonio Assalini, Kazuhiro Ema, and Masahide Sasaki. Displacement receiver for phase-shift-keyed coherent states. Phys. Rev. A, 86:042328, 2012.
20. R. S. Kennedy. A near-optimum receiver for the binary coherent state quantum channel. Technical report, Massachusetts Institute of Technology (MIT), 1973.
21. S. J. Dolinar. An optimum receiver for the binary coherent state quantum channel. Technical report, Massachusetts Institute of Technology (MIT), 1973.
22. Christoffer Wittmann, Masahiro Takeoka, Katiúscia N. Cassemiro, Masahide Sasaki, Gerd Leuchs, and Ulrik L. Andersen. Demonstration of near-optimal discrimination of optical coherent states. Phys. Rev. Lett., 101:210501, 2008.
23. F. E. Becerra, J. Fan, G. Baumgartner, S. V. Polyakov, J. Goldhar, J. T. Kosloski, and A. Migdall. m-ary- state phase-shift-keying discrimination below the homodyne limit. Phys. Rev. A, 84:062324, Dec 2011.
24. Michele N. Notarnicola, Matteo G. A. Paris, and Stefano Olivares. Hybrid near-optimum binary receiver with realistic photon-number-resolving detectors. J. Opt. Soc. Am. B, 40(4):705–714, 2023.
25. Kenji Tsujino, Daiji Fukuda, Go Fujii, Shuichiro Inoue, Mikio Fujiwara, Masahiro Takeoka, and Masahide Sasaki. Quantum receiver beyond the standard quantum limit of coherent optical communication. Phys. Rev. Lett., 106:250503, Jun 2011.